A CERTAIN CLASS OF EXACT SOLUTIONS OF THE EQUATIONS OF MAGNETO-GASDYNAMICS

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The equations which characterize the nonsteady gas flow in a magnetic field for gases having infinitely large conductivity, for the case of conical symmetry, have the form:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial}{\partial r}\left(p + \frac{h^2}{2}\right) + \frac{mh^2}{\rho r} = 0 \quad \left(h = \frac{H}{\sqrt{4\pi}}\right) \tag{1}$$

$$\frac{\partial \ln \rho}{\partial t} + u\frac{\partial \ln \rho}{\partial r} + \frac{\partial u}{\partial r} + \frac{Nu}{r} = 0 \tag{2}$$

$$\frac{\partial \ln p}{\partial t} + u\frac{\partial \ln p}{\partial r} + k\frac{\partial u}{\partial r} + kN\,\frac{u}{r} = 0 \tag{3}$$

$$\frac{\partial \ln h}{\partial t} + u\frac{\partial \ln h}{\partial r} + \frac{\partial u}{\partial r} + (N - m)\frac{u}{r} = 0 \tag{4}$$

where u is the gas velocity, ρ the density, p the pressure, H the intensity of the magnetic field, always perpendicular to the velocity of gas flow, k the ratio of specific heats, $N = 0$, $m = 0$ for one-dimensional flow; $N = 1$, $m = 0$ for flow with cylindrical symmetry, when $H = H_z(r, t)$ and $N = 1$, $m = 0$ for the flow with cylindrical symmetry, when $H = H_\phi(r, t)$.

Let the pressure depend only on time $p = p(t)$, while the field intensity

$$H = \sqrt{4\pi}\, r^{-m} f(t) \quad \text{or} \quad h = r^{-m} f(t) \tag{5}$$

Substituting these values p and h into equation (1), we obtain

$$r = ut + F(u) \tag{6}$$

Introducing new independent variable t, u in place of t, r and taking equation (6) into account, we obtain

$$\rho = \varphi_1(u)\, r^{-N}(t + F')^{-1}, \qquad p = \varphi_2(u)\, r^{-kN}(t + F')^{-k}, \qquad h = \varphi_3(u)\, r^{-(N+m)}(t + F')^{-1}$$

In order to meet the requirement $p = p(t)$ and $h = r^{-m} - f(t)$, it is necessary to assume that

$$F(u) = -ut_0, \qquad \varphi_2 = Au^{kN}, \qquad \varphi_3 = Bu^N$$

Then the solution of the main system will have the form:

$$\rho = \varphi_1(u)(t - t_0)^{-(N+1)}, \qquad p = A(t - t_0)^{-k(N+1)}, \qquad h = Br^{-m}(t - t_0)^{-(N+1)}$$

where A and B are constants.

Translated by J.R.W.